Accumulation, storage and manipulation of large numbers of positrons in traps I. — The basics

C. M. Surko

University of California - San Diego, La Jolla CA 92093, USA

Summary. — Methods are described to create, store, manipulate and characterize positron plasmas. Emphasis is placed on the so-called buffer-gas positron trapping scheme for positron accumulation that uses positron-molecule collisions to accumulate particles efficiently. Manipulation and storage techniques are described that exploit use of the Penning-Malmberg trap, namely a uniform magnetic field with electrostatic confining potentials along the direction of the field. The techniques described here rely heavily on single-component-plasma research, and relevant connections are discussed. The use of rotating electric fields to compress plasmas radially (the so-called “rotating wall” technique) is described; it has proven particularly useful in tailoring positron plasmas for a range of applications. The roles of plasma transport and available cooling mechanisms in determining the maximum achievable plasma density and the minimum achievable plasma temperature are discussed. Open questions for future research are briefly mentioned.

1. — Overview

The school on the physics with many positrons, which sparked the writing of this volume, highlighted the fact that progress in the ability to accumulate and cool positrons and antiprotons is enabling new scientific and technological opportunities with low-energy antimatter. In some sense, much of the driver for this work is forefront plasma physics
research — developing new ways to create and manipulate antimatter plasmas. This and a companion paper, denoted here as paper II [1], describe the development of new plasma tools for this effort. The emphasis is on methods to efficiently accumulate and store positrons in the form of single-component plasmas and to compress them radially using rotating electric fields. Aspects of these techniques are also relevant to the confinement and manipulation of antiprotons(1).

Paper II describes recently developed methods to create positron beams with small transverse spatial extent. The prospects for accumulating and storing larger quantities of antimatter are also discussed in paper II, namely a novel multi-cell positron trap capable of storing $\geq 10^{12}$ positrons for days or longer, as well as other selected topics. These chapters are intended to be tutorial in nature rather than a first description of research results. They borrow heavily from previously published material, sometimes repeating passages verbatim. This chapter relies heavily on the material in refs. [2-4]. The reader is requested to consult these and other original articles for further details.

Single-component plasmas are the method of choice to accumulate, cool, and manipulate large numbers of antiparticles. These collections of antimatter can be stored in a high quality vacuum for very long times using the suitably arranged electric and magnetic fields of a Penning-Malmberg trap [5] — this device functions as a nearly ideal electromagnetic bottle. Not only can these positron plasmas be made more or less arbitrarily free of annihilation, but in addition, techniques are available to further cool, compress, and tailor them for specific applications. These antimatter plasmas now play an important role in science and technology and this is expected to continue.

Low-energy antimatter science relies upon many developments in positron technology. They include methods to cool plasmas rapidly using specially chosen buffer gases [6] or cyclotron emission in a large magnetic field [7]; the application of rotating electric fields for radial plasma compression [3,4,6,8-10]; the development of non-destructive diagnostics using plasma waves [10-12]; and the creation of beams of small transverse spatial extent by careful extraction from trapped and cooled antimatter plasmas [13-15].

There are numerous applications of these positron plasmas and trap-based beams. Trapped positron plasmas and similarly confined clouds of antiprotons are the method of choice to make low-energy antihydrogen atoms [10,16-19]. One goal of that work is to test fundamental symmetries of nature by precision comparisons of hydrogen and antihydrogen. Attempts are being made to create and study electron-positron plasmas that are of interest in plasma physics and astrophysics [20-23]. Bursts of positrons from a trap-based beam were used to create the first positronium molecules $(Ps_2)$. This represents an important step toward the creation of a Bose-Einstein condensate (BEC) of Ps atoms [24]. Positrons have been used extensively to study materials [25-27], such as low dielectric constant insulators that are key components in high-speed electronics and

(1) There are, however, significant differences. Due to antiproton’s annihilation characteristics and heavier mass, the positron cooling techniques described here must be replaced, cooling the antiprotons sympathetically with cold electrons. Further, the accumulations of antiprotons to date have typically been gases of charged particles rather than plasmas.
chip manufacture [27]. An important focus of recent work is the further development of pulsed, trap-based positron beams that offer improved methods to make a variety of materials measurements. Commercial prototypes of these beam systems are now available [28,29]. Positrons are also important in medicine and biology; positron emission tomography is the method of choice to study metabolic processes in humans and animals, both to treat disease and to develop new therapies [30]. In the longer term, research in this area may well lead to the development of portable antimatter traps, and this, in turn, would facilitate many other uses of antimatter [2,31].

Much of the following discussion relies on the physics of single-component plasmas in Penning-Malmberg (PM) traps, namely a plasma in a cylindrical set of electrodes immersed in a uniform magnetic field with electrostatic confinement along the direction of the field. Relevant parameters to describe these plasmas and the notation used here and in paper II are listed in table I(2). The book by Davidson [32] and the review article by Dubin and O’Neil [33] contain excellent, detailed discussions of the theoretical plasma physics concepts relevant to non-neutral plasmas, including those in PM traps.

(2) Expressions in this paper are in S. I. units, unless otherwise noted. In these units, $\varepsilon_0$ is the permittivity of free space.
2. – Positron trapping

2'1. Background and overview. – In our world of matter positrons are typically produced using accelerators or radioisotopes. To be trapped, they must be slowed to electron-volt energies from their initial, broad spectrum of energies, ranging from several kiloelectron volts to $\sim 0.5\,\text{MeV}$. Typically a "moderator" material is used to slow them down. This is done by either transmission through, or reflection from a metal, such as single-crystal copper or tungsten (energy spread $\sim 0.5\,\text{eV}$; efficiency $\leq 0.1\%$) [25, 34], or reflection from a frozen, solid rare gas such as neon (energy spread $\sim 1\,\text{eV}$; efficiency $\geq 1\%$) [35, 36]. These materials are chosen specifically for the characteristic that positrons do not readily bind to them or become trapped in voids or at defects. In particular, some metals have a negative positron work function and can be grown in large single crystals; they are thus well suited for positron moderation.

The accumulation and confinement of positrons in electromagnetic traps has a long history. In the early 1960s, Gibson, Jordan and Lauer injected radioactive neon gas in a vacuum chamber surrounded by magnetic mirror coils [37]. The emitted positrons were confined by the mirror field. The escape time, relative to the Ne gas puff, was used to measure the single-particle confinement time. Schwinberg, Van Dyck and Dehmelt confined small numbers of positrons in a Penning trap for very long times (weeks to months) [38]. Their goal was to make precision comparisons of the properties of electrons and positrons. Mills and collaborators used a Penning trap to confine and bunch positrons from a radioisotope source [39] and from a microtron accelerator [40] for use in spectroscopic studies of Ps atoms. Brown, Leventhal, Mills, and Gidley confined positrons in a Penning trap to measure the annihilation Doppler broadening spectrum of molecular hydrogen in order to model astrophysical annihilation spectra [41]. In all of these experiments, small numbers of positrons were confined at low densities (i.e., typically in the positron-gas regime rather than the plasma regime). Here we focus on the accumulation of large numbers of positrons in the plasma regime.

While a number of devices and protocols have been used or proposed to trap antimatter, the device of choice is the PM trap because of its excellent confinement properties. Other variations of the Penning trap that have either been discussed or employed to trap antiparticles and antimatter plasmas include hyperboloidal [38], orthogonalized cylindrical [42] and multi-ring electrode structures [43]. The PM trap is illustrated in fig. 1. It uses a uniform magnetic field to inhibit the diffusion of particles across the $B$ field and an electrostatic potential well, imposed by the application of suitable voltages on a set of cylindrical electrodes, to confine the particles in the direction of the $B$ field [33, 44, 45]. The extremely long confinement times that can be achieved in these traps [5, 46] makes the accumulation of substantial amounts of antimatter in the laboratory feasible.

2'2. An efficient accumulation scheme: the buffer gas trap. – Given this good trapping scheme, the challenge is to find an efficient method to fill the trap with positrons. A variety of trapping techniques have been developed to do this. If a pulsed positron source such as a LINAC is used, the positrons can be captured by timed switching of the potential on
Fig. 1. – Schematic diagram of a PM trap. The plasma is shown in the cut-away section. It is confined by a uniform axial magnetic field and by the electrostatic potential, $V$, at each end. As described below, the plasma rotates about its axis with an $E \times B$ frequency $f_E$, as illustrated in the end-on view (right).

one of the end confining electrodes. This end-gate switching technique has been employed extensively to condition positron beams from LINACs and other pulsed sources [47]. It has also been used to transfer positrons from one trap to another [19,48,49]. For high capture efficiency, the spatial extent of the incoming pulse must be smaller than twice the trap length, and the slew rate on the capture gate must be sufficiently rapid. In many circumstances, these conditions are relatively easy to meet.

When positrons are captured from a steady-state source, such as a radioisotope, energy can be extracted from the positrons to trap them, or energy can be transferred from the positron motion in the direction parallel to the magnetic field to the perpendicular direction by a variety of techniques. The latter effect results in “virtual trapping” in that the particles can subsequently be de-trapped by the reverse process. A variety of techniques have been developed to trap positrons using these approaches, including collisions with neutral gas atoms and molecules [50,51], scattering from trapped ions [38,52], scattering from trapped electrons in a nested potential well [53], and trapping in a magnetic mirror [54]. Other methods used to trap positrons include using dissipation in an external resistor [55], field ionization of weakly bound positronium atoms [52,56], and the exchange of parallel and perpendicular momentum exploiting stochastic orbits [57]. Each of these techniques has its advantages, but it has turned out that they are relatively inefficient.

The positron trapping method most widely used is the buffer gas (BG) technique. It has the highest trapping efficiency and modest magnetic-field requirements. Figure 2 illustrates the operating principle of such a buffer gas positron accumulator which, in this example, has three stages [51,58]. Figure 3 shows the actual physical arrangement. Positrons are injected into a specially modified PM trap having a stepped potential profile, with each stage having a different pressure of buffer gas. Using a continuous gas feed and differential pumping, a high pressure ($\sim 10^{-3}$ torr) is maintained in the small-diameter region at the left (“stage I”). Positrons are initially trapped in this region by inelastic collisions with buffer gas molecules (marked “A” in the figure). The trapped positrons then make multiple passes back and forth in the trap. They lose energy by subsequent inelastic collisions (“B” and “C”) in the successively lower pressure stages II and III, causing them to accumulate in stage III. Here, they cool to approximately the gas (i.e., the electrode) temperature, which is $\sim 300$ K.
Fig. 2. – Buffer gas trapping scheme, showing the electrode geometry of a modified PM trap (above), the neutral gas pressure in each stage, and the axial potential profile (below). There is an applied magnetic field, $B \sim 0.15 \text{T}$, in the $z$-direction. Two-stage accumulators with $B$ as small as $0.04 \text{T}$ have also been used successfully [59].

Fig. 3. – A three-stage positron trapping apparatus with source and moderator at UCSD. Above: cutaway of a three-stage positron trap; and below: photograph of the positron source (a sealed $^{22}\text{Na}$ radioisotope source and solid neon moderator) in a lead enclosure at the left, and the three-stage trap in the large metal box on the right. For spatial scale, the floor tiles are $\sim 0.3 \times 0.3 \text{m}$. 
Fig. 4. – Cross-sections in atomic units ($a_0^2 = 2.8 \times 10^{-21} \text{m}^2$) for positron-impact excitation of the $a^1\Pi$ electronic state of $N_2$ (\textgreater) and positronium formation (\textbullet). The dashed and solid vertical bars indicate the thresholds for electronic excitation and Ps formation, respectively. From ref. [60].

This type of accumulator can be operated using a variety of gases including molecular nitrogen, hydrogen, carbon dioxide and carbon monoxide [28]. There are two considerations regarding the choice of buffer gas. One is to find a target species that has a relatively large cross-section for energy loss via inelastic scattering. The second is to avoid positronium (Ps) atom formation, which results in loss of positrons through annihilation, either in the Ps atom or when the Ps strikes an electrode or the vacuum chamber.

It would be appealing to use the vibrational excitation of molecules for this energy loss process, however this results in a loss per collision $\leq 0.5 \text{eV}$. In practice, this is too small to efficiently trap the spread of positrons from the moderator (e.g., energy spreads $\sim 1 \text{eV}$). An important effect is due to the fact that typical source/moderators are operated at a reduced magnetic field (typically $B \leq 0.03 \text{T}$). The quantity $E_\perp/B$ is an adiabatic invariant for these particles, where $E_\perp$ is the energy in motion in the plane perpendicular to $B$. Thus, when particles with a spread of $E_\perp$ values enter the higher magnetic field of the BG trap, the spread in parallel energies, $E_\parallel$, increases significantly. This generally reduces the trapping efficiency since the inlet potential cannot be as carefully tuned so that incoming positrons just pass over it. The entire spread of $E_\parallel$ must now pass over the inlet potential barrier of the trap. Positrons with larger values of $E_\parallel$ must lose correspondingly more energy before they become trapped, and it is more difficult to tune the potential steps to optimize the energy loss per collision for all of the particles.

The highest trapping efficiency is obtained using molecular nitrogen. The reason it is superior is that, as shown in fig. 4, this species has a relatively large electronic excitation cross-section at positron impact energies $\sim 10 \text{eV}$, near the threshold for electronic excitation of the $a^1\Pi$ of $N_2$ at 8.8 eV [60], while it also has a relatively small cross-section for Ps formation (i.e., a potent positron loss process) in this range of energies. To our
knowledge, molecular nitrogen is somewhat unique in this important characteristic. In most other molecules, the Ps formation threshold is below that for the lowest allowed inelastic electronic transition.

The pressure in stage I is set so that the positrons make, on average, \( \sim \) one electronic-excitation collision in one transit through the trap and hence are confined in the potential well. This happens before they reflect off the potential barrier at the end of the trap opposite the source, exit the trap, and return to the moderator, where they would be lost to annihilation. Once trapped, the positrons move back and forth in the direction of the magnetic field. Additional stages with stepped potentials and correspondingly lower neutral gas pressures (\( i.e., \) two more stages in the trap illustrated in fig. 2) are arranged to trap the positrons in a region of low gas pressure in which the annihilation time is commensurately long. The positron lifetime in stage III of the trap illustrated in fig. 2 is typically \( \geq 40 \) s. Longer lifetimes (\( e.g., \) hours or more) can be achieved by pumping out the buffer gas following positron accumulation.

While \( N_2 \) has a relatively large electronic excitation cross-section, its vibrational excitation cross-section is quite small. The addition of a low pressure (\( e.g., \) \( \leq 10^{-7} \) torr) of \( CF_4 \) or \( SF_6 \) in stage III is used to cool rapidly to room temperature [8]. The unusually large positron-impact vibrational cross-section of carbon tetrafluoride [61], which is discussed in more detail below, is responsible for rapid cooling to temperatures \( \leq 0.16 \) eV, and \( SF_6 \) is believed to act similarly. For the typical pressure settings in the three-stage trap shown in fig. 3, operating with \( N_2 \) in stages I-III and \( CF_4 \) in stage III, the positrons are trapped in one transit back and forth through the trap. They lose additional energy by a second electronic excitation of \( N_2 \) and are thus confined in stages II and III in \( \leq 100 \) \( \mu \)s. The positrons then make a similar collision in stage III and are confined to this stage in a few ms [51]. Finally, the positrons cool to room temperature by vibrational and rotational excitation of \( CF_4 \) in \( \leq 0.1 \) s. A set of rate equations describing this cascade to lower positron energies is discussed in ref. [51].

For accumulators with a solid neon moderator, the trapping efficiencies (\( i.e., \) defined as the fraction of positrons trapped and cooled relative to the number of incident slow positrons from the moderator) are typically in the range of 5–20\%, and efficiencies of up to 30\% have been observed under optimized conditions. Using a tungsten moderator, the efficiency can be as high as 50\%. While not studied in detail, the trapping efficiency is likely limited by positronium atom formation and the small positron density in the first stage of the trap. This positron-density effect, which is discussed in more detail below, is due to \( \vec{E} \times \vec{B} \), asymmetry-induced radial transport, where \( \vec{E} \) is the (dc in the laboratory frame) electrostatic field due to trap asymmetries. It is largest in the first trapping stage where the positron density is the smallest.

Using a 100 mCi \(^{22}\)Na source and solid neon moderator, several hundred million positrons can be accumulated in a few minutes in the three-stage trap shown in fig. 3 [62]. Once accumulated, the resulting positron plasmas can be transferred efficiently to another trap and stacked (\( e.g., \) for long-term storage) [19,48,49]. Figure 5 shows the history of positron trapping using an apparatus such as that described here using similar strength sources.
Fig. 5. – Progress in creating positron gases and plasmas in PM traps using $^{22}$Na positron sources with strengths $\sim 50$–100 mCi. For the data before 1993, tungsten moderators were used, while after that, solid neon moderators were used.

These buffer gas traps are relatively efficient, arguably even efficient on an absolute scale. The difference between 5% and 30% efficiency is typically due to the fine-tuning of the alignment of the incoming positron beam with respect to the electrode structure. In this regard, careful choice of the inner-diameter of the first stage electrodes and operating pressure is likely of considerable importance. One area that, to the author’s knowledge, has not been explored extensively is the extent to which elastic scattering on atoms or molecules (i.e., transfer of parallel energy to that perpendicular to the $B$ field), and the resulting process of virtual trapping could be used to advantage, particularly in the first stage of the buffer gas trap. In this process, the particles will be trapped until another elastic scatter de-traps them, or an inelastic collision traps them absolutely.

Simpler, two-stage positron accumulators with correspondingly shorter positron lifetimes (e.g., $\leq 1$ s) have now been developed [59,63]. Commercial two- and three-stage positron traps, such as that shown in fig. 6, are now sold commercially by R. G. Greaves at First Point Scientific, Inc., Agoura Hills CA.

We end this discussion with a cautionary practical note about positron traps such as those described here. It is well known that positron annihilation rates on large hydrocarbon molecules can be extremely high. This arises from the fact that positrons tend to bind to these species (i.e., through a mechanism known as vibrational Feshbach resonances) [64]. Oil molecules are particularly deleterious in this regard. Thus considerable care must be taken in achieving a good, oil-contaminant-free base vacuum in the accumulator (e.g., $\leq 5 \times 10^{-10}$ torr) and/or trap. The vacuum system should be bakable (e.g., to 420 K or higher), if long confinement times are desired.

3. – Positron cooling

Moderator materials (as described above) are used to decelerate high-energy positrons from a source to electron-Volt energies. Once accumulated in a PM trap, these collections
Fig. 6. – A commercial two-stage buffer gas trap and a separate storage stage (i.e., a three-stage system) with a $^{22}$Na source and solid neon moderator. Photograph of the system (below) with, left to right, the source/moderator (in the shiny cylinder), then (in the black solenoids) the buffer gas trap and the storage stage. Also shown are (left top) the buffer gas trap electrodes, and (right top) the storage stage electrodes. Courtesy of R. G. Greaves, First Point Scientific, Inc., Agoura Hills CA.

of charged particles can be heated by small electric perturbations. This is quite deleterious to positron confinement for a number of reasons. For example, heating to above the energy threshold for Ps formation leads to a first-order particle loss. Furthermore, increased positron energy can also lead to de-confinement. Thus arranging an effective method to cool these positron gases and plasmas is extremely important. This is absolutely obligatory in cases where the plasma can be heated substantially. Such heating can occur, for example, when rotating electric fields are used to compress plasmas radially, or when there is a manipulation of positron gases and plasmas between various trapping regions.

31. **Collisional cooling using atomic or molecular gases.** – At electron-Volt energies and below, positron cooling can be accomplished by collisions with suitable gases of atoms or molecules. This was described briefly above, but a bit more amplification is in order. The cooling gas is selected to have a large inelastic scattering cross-section to achieve significant energy loss. However positron loss due to positronium (Ps) atom formation must be avoided if possible. So-called “direct” annihilation of a positron with a bound electron in an otherwise elastic collision typically has a much smaller cross-section. Thus,
TABLE II. – Positron cooling rates in a PM trap using molecular gases at 2 × 10⁻⁸ torr. Time $\tau_a$, for direct annihilation; measured cooling time, $\tau_c$; and the energies of the vibrational quanta, $\varepsilon_j$. Data from refs. [6,8].

<table>
<thead>
<tr>
<th>Gas</th>
<th>$\tau_a$ (10³ s)</th>
<th>$\tau_c$ (s)</th>
<th>$\varepsilon_j$ (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF₆</td>
<td>2.2</td>
<td>0.36</td>
<td>0.076, 0.19</td>
</tr>
<tr>
<td>CF₄</td>
<td>3.5</td>
<td>1.2</td>
<td>0.16</td>
</tr>
<tr>
<td>CO₂</td>
<td>3.5</td>
<td>1.3</td>
<td>0.29, 0.083</td>
</tr>
<tr>
<td>CO</td>
<td>2.4</td>
<td>2.1</td>
<td>0.27</td>
</tr>
<tr>
<td>N₂</td>
<td>6.3</td>
<td>115</td>
<td>0.29</td>
</tr>
</tbody>
</table>

where possible, one tries to work below the threshold for Ps formation (i.e., which can be several electron-Volts or more). In fact, to avoid loss due to Ps formation with positrons on the tail of the Maxwellian distribution, the positron temperature should be kept a factor of three or more below the Ps formation threshold (e.g., $T \leq 2$ eV). For relatively low positron temperatures, direct annihilation then becomes an important factor in determining the lifetime of trapped positrons (e.g., $\sim 40$ s for N₂ at a pressure $\sim 10^{-6}$ torr).

Only recently have state-resolved inelastic positron-impact cross-sections been measured [64]; and so a general, quantitative understanding of the collisional positron cooling processes involving atoms and molecules is not available. Typically at energies in the electron-Volt range, electronic transitions can be used to reduce the positron energy effectively. At energies in the 0.05 to several eV range, vibrational transitions in molecules can be used, while below $\sim 0.05$ eV, one must rely on rotational transitions in molecules and momentum transfer collisions with atoms to cool the positrons.

In the case where a single inelastic scattering channel is relevant (e.g., a vibrational mode $j$ with energy $\varepsilon_j$), the cooling rate $\Gamma_c$ will be

\[ \Gamma_c \equiv \frac{1}{T} \frac{dT}{dt} \approx -\frac{\nu_j \varepsilon_j}{T}, \]

where $\nu_j$ is the excitation rate for this transition. As we discuss in more detail below, this collisional cooling rate is applicable if there are no heating electric fields. If there are heating fields, then any momentum transfer (e.g., an elastic) collision can convert the coherent field-driven component of the kinetic energy into heat, and this must be taken into account to determine the net cooling/heating rate. Such a detailed account of scattering processes is beyond the scope of this review. Likely Monte Carlo computer calculations would be useful in studying this balance of heating and cooling [65], assuming the necessary collisional cross-section data are available.

Cross-sections for vibrational excitation of molecules have now been measured for several species [66,67], and at least a semi-quantitative understanding of the magnitudes of these cross-sections is available [68]. Moreover, positron cooling rates due to vibrational excitation have been measured for several molecules [6,28,69]. Cooling rates for selected molecules are given in table II. It turns out that SF₆ and CF₄ are particularly effective.
Fig. 7. – Positron impact cross-section for excitation of the $\nu_3$ vibrational mode of CF$_4$ as a function of the incident positron energy in atomic units ($a_0^2 = 2.8 \times 10^{-21}$ m$^2$). The relatively large and approximately constant cross-section above the threshold energy, $\varepsilon_j = 0.157$ eV, provides a very efficient and useful cooling mechanism. Reprinted from ref. [61].

In these species, there is a large amount of charge transfer to the F atoms. This results in a very large vibrational excitation cross-section for the asymmetric stretch ($i.e.$, $\nu_3$) vibration in the molecule. Shown in fig. 7 is the cross-section for the $\nu_3$ mode of CF$_4$ [61].

The current versions of buffer gas positron traps typically use a mixture of N$_2$ and CF$_4$ or SF$_6$ in the final trapping stage for rapid cooling. There is very little information available on positron energy loss due to rotational excitation of molecules, save for an early study by Coleman et al., using a positron lifetime technique [70], thus this would likely be a fruitful area for future work. As discussed below, CF$_4$ has also been used effectively for the cooling required for the radial compression of positron plasmas ($i.e.$, to counteract the heating caused by the work done on the plasma by the applied torque).

3.2. Cyclotron cooling. – A convenient method to cool electron mass charged particles (positrons) in an ultra-high vacuum (UHV), is to arrange for them to emit cyclotron radiation in a strong magnetic field [46]. In this case, the positron temperature is typically a balance of heating ($e.g.$, rf electric fields are particularly effective in this regard) and the cyclotron cooling. In fact, cyclotron cooling at achievable magnetic fields is typically considerably less effective than cooling using gaseous collisions; so in this case, heat sources can produce quite significant effects. In the absence of a heating source, the particles will come to equilibrium at the temperature of the surrounding electrode structure. However, in the case in which parts of the vacuum system are at higher temperatures ($e.g.$, when the electrodes are cooled cryogenically), this can result in heating
the plasma above the temperature of the electrode structure\(^{3}\). The cyclotron cooling rate for electron mass charged particles is \([46, 71]\)

\[
\Gamma_c \approx B^2 / 4,
\]

where \(B\) is in teslas and \(\Gamma_c\) is in \(s^{-1}\). For example, the radiative cooling time, \(1/\Gamma_c\) of positrons in a 5 T field is 0.16 s. Assuming an emissivity, \(\varepsilon = 1\), for the electrodes at the cyclotron frequency, the surrounding electrode structure is at temperature \(T_w\), and there is no external heating, the time dependence of the positron temperature, \(T(t)\), of a positron plasma at initial temperature \(T_1\) will be

\[
T(t) = T_0 + (T_1 - T_w) \exp(-\Gamma_c t).
\]

Shown in fig. 8 is a typical cooling curve for the thermal relaxation of an electron plasma confined in an apparatus at 300 K.

Two comments are in order regarding cyclotron cooling in an electrode structure. If one can arrange a resonant cavity at the cyclotron frequency, then the cooling rate is increased by the \(Q\) factor of the cavity [72]. The second comment is that the electrode structure must have a minimum size in order for cyclotron cooling to be effective. In particular, the structure must be at least large enough to accommodate the lowest-order resonant mode. For a long circular electrode structure, this means that the inner diameter of the structure must be \(D \geq \lambda_c\), where \(\lambda_c\) is the electromagnetic wavelength at the cyclotron frequency. For smaller values of \(D\), the electrodes will act as a waveguide beyond cutoff and radiation by the particles will be suppressed.

\(^{3}\) J. Fajans, private communication, 2009.
3.3. Sympathetic cooling using ions. – The techniques described above are limited to producing a temperature equal to the temperature of the environment, (e.g., 4 K for cyclotron cooling in a trap cooled to liquid-helium temperature). However, laser cooling of ions in traps permits cooling to temperatures much lower than their surroundings. This technique has been used \([52]\) to reach positron plasma temperatures significantly below the ambient by cooling the positrons sympathetically using laser-cooled ions that were simultaneously confined in the same trap with the positron plasma. Using this technique, a high-density positron plasma \((n = 4 \times 10^{15} \text{ m}^{-3})\) was cooled to \(< 5 \text{ K}\) in a room temperature trap. This technique has the potential to produce positron plasmas with parallel energies less than 100 mK, see footnote \((4)\).

4. – Confinement and characterization of positron plasmas in Penning-Malmberg traps

4.1. Basic concepts. – A typical PM trap for positrons is shown in fig. 9. It consists of a set of cylindrical electrodes in a uniform magnetic field. The plasma is confined in the direction of the magnetic field by electrostatic potentials applied to electrodes at each end. A segmented electrode over a portion of the plasma is used to apply a rotating electric field to compress the plasma radially (i.e., this is discussed in more detail in

\((4)\) At very large \(B\) fields and low temperatures, the perpendicular energy of the particles will eventually be limited by the energy of the lowest Landau level.
sect. 5, below). Also shown is a phosphor screen and CCD camera for imaging the radial
distribution of the plasma [6] and the rf circuitry to excite waves in the plasma (e.g., for
temperature and density measurements) [73, 74].

In a single-component plasma at temperature $T$ in the PM trap, the particles make
only small excursions in the plane perpendicular to $B$. They are characterized by the
(average) cyclotron radius, $r_c = v_T / \omega_c$, where the cyclotron frequency $\omega_c$ is the angular
frequency of gyration of the particle in the plane perpendicular to $B$, and $v_T = (T/m)^{1/2}$
is the average thermal velocity of the particle.

They are subject to a confinement principle that arises from the fact that a charged
particle in a $B$ field has an angular momentum associated with it, beyond the ordi-
nary mechanical momentum. As a consequence, at low temperatures where the thermal
velocities of the particles are negligible, the canonical angular momentum $P_\theta$ [46] is
approximately

$$P_\theta \approx -\frac{m\omega_c}{2} \sum r_i^2,$$

where the $r_i$ are the radial positions of the particles, and it is assumed that the particles
are positively charged which fixes the sign of $P_\theta$.

In a PM trap with cylindrically symmetric electrodes, the angular momentum, $P_\theta$, is
constant. Thus the second radial moment of the particle distribution is also constant,
and so the plasma cannot expand. In practice, these plasmas do expand slowly due to
imperfections in the trap. In this case, the torque on the plasma is related to the outward
transport rate, $\Gamma_0 = (1/n)(dn/dt)$, by

$$\tau = \frac{dP_\theta}{dt} = P_\theta \Gamma_0.$$

A single-component plasma in a PM trap is effectively a long cylindrical rod of charge.
This collection of trapped particles will exhibit plasma behavior when the Debye length,
$\lambda_D = v_{th}/\omega_p$, is such that $\lambda_D \gg r_p, L_p$, where $\omega_p$ is the plasma frequency, $r_p$ is the
plasma radius, and $L$ the plasma length. In this case, potential perturbations in the
plasma will be screened by the motion of the particles in the direction parallel to $B$.
Consequently, any remaining electric field in the plasma will be in the radial direction
(i.e., neglecting end effects).

This radial electric field in and around the plasma results in a plasma potential that
increases as one approaches the plasma center. From Gauss’ law, for a long cylindrical
plasma of $N$ positrons with radius $r_p$ in an electrode of radius $r_W$, the magnitude of this
space charge potential (i.e., the “plasma potential”) at the plasma center is

$$\Phi = \frac{AN}{L} [1 + 2 \ln(r_W/r_p)],$$

where $A = e/4\pi \varepsilon_0 = 1.4 \times 10^{-9}$ V m. This value of $\Phi$ sets the minimum potential, $V_c$,
required to confine the plasma, namely $V_c > \Phi$. 
Fig. 10. \(E \times B\) drift orbits for 5 cyclotron periods in a two-dimensional “slab” model: (a) the usual drift-orbit case where the electric field is small, \(r_E/r_c = 0.1\), and (b) a much larger electric field, \(r_E/r_c = 10\). The corresponding radial excursions \(\rho\) and lateral distances \((D)\) are approximately (a) \(r_c (5r_c)\), and (b) \(10r_c (50r_c)\). The particles make much larger excursions in the strong \(E\) field. (In the PM trap, \(y\) corresponds to the radial direction and \(x\) the azimuthal direction.)

A key physical effect in PM traps and in other magnetized plasmas arises from the fact that the magnetic and electric space charge fields are perpendicular to each other. This is illustrated in fig. 10 for a “slab” model that describes particle motion in two dimensions (i.e., ignoring the cylindrical symmetry of the PM trap). Charged particles in such fields undergo so-called “\(E \times B\) drifts” at a velocity \(v_E = E/B\), in the direction perpendicular to both \(E\) and \(B\) [75, 76]. In terms of the cyclotron radius \(r_c\) and the characteristic distance \(r_E = v_E/\omega_c\), the trajectories are “cycloids.” The particles orbit about a center moving at velocity \(v_E\), located at an \((x, y)\) position \((r_E, r_c + v_E t)\), with an associated radius,

\[
\rho = \sqrt{r_c^2 + r_E^2}.
\]

In particular, the secular motion is in the \(x\) direction (i.e., perpendicular to both \(E\) and \(B\)), and the oscillation amplitude \(2\rho\) is dominated by the larger of \(r_E\) and \(r_c\). This latter effect has very important consequences for particle transport, namely the transport step size (\(\sim \rho\)) can be dominated by \(r_E\). Thus the transport can become very large when \(E\) is large (i.e., \(v_E > v_T\)) giving rise to large and rapid excursions of the particles outward.
Due to the radial $E$ field in the trapped plasma and the resulting $E \times B$ motion, the particles drift around the axis of symmetry at a frequency

$$f_E = \frac{ne}{4\pi \varepsilon_0 B},$$

where $n$ is the number density of the plasma. Note here the intimate connection between the rotation frequency, $f_E$, and the plasma density, $n$.

If the rotation frequency is not constant as a function of radial position, there will be a shear on the charged fluid, and the viscosity will act to oppose this shear flow. Quite generally, if there are minimal torques on the plasma and no strong heating sources, the plasma will tend to assume a state that approximates closely the shear-free, constant-density rigid rotor in thermal equilibrium at some temperature $T$ [77]. This model is frequently applicable, and it results in an enormous simplification in describing the behavior of single-component plasmas.

4.2. Transport due to neutral collisions. – The angular momentum constraint of eq. (4) implies that a single-component plasma confined by a magnetic field can expand only if there is a torque on it. In a perfect, azimuthally symmetric trap there would be no expansion. However in practice, this is not the case. Typically radial transport is observed. This can be due to trap imperfections; or in the case where there is appreciable neutral background gas, this transport can be due to the drag on the plasma due to neutral collisions [5, 78].

The transport due to neutral gas collisions is reasonably well understood. In this case, the outward flux of particles (i.e., number of particles/area-time) $J$ is [5]

$$J = \frac{\partial}{\partial r} (\nu_p r_c^2 n) + \nu_p r_c^2 \left( \frac{eE}{T} \right) n,$$

where $\nu_p$ is the positron-neutral, momentum transfer collision frequency (frequently dominated by elastic scattering), and $E$ is the space charge electric field. The two terms in eq. (9) are, respectively, the flux due to collisional diffusion, and the flux induced by the electric field that involves the electrical mobility of the plasma. In the plasma regime, $(eE r_p)/T \gg 1$, and so the second term dominates the otherwise diffusive transport by a factor $\sim e\Delta \Phi/kT$, where $\Delta \Phi$ is the change in plasma potential across the plasma [5]. Assuming this is the case and inserting $E$ for a rigid rotor, one finds for the outward transport rate, $\Gamma_0 \equiv (1/n)(dn/dt)$,

$$\Gamma_0 = \nu_p \left( \frac{r_c}{\lambda_D} \right)^2.$$

In the single-component positron (or electron) plasmas considered here, typically $r_c \ll \lambda_D$, and so the transport due to neutral-gas collisions is typically small.
Fig. 11. – The expansion rate, $\Gamma_0$, as a function plasma density for an electron plasma in a UHV PM trap in a 5 T field. The data show two regimes, including one in which $\Gamma_0$ is strongly density dependent. The transition occurs when the Coulomb collision frequency is $\sim$ three times the axial bounce frequency. (In this figure, $L \equiv L_p$) From ref. [80]; see this reference for details.

4.3. Transport due to electric and magnetic asymmetries. – In the case that gas collisions do not dominate the transport (e.g., a plasma in a UHV environment cooled by cyclotron radiation), a detailed, microscopic understanding of the transport has remained elusive in spite of 30 years research on the subject. It is believed to be due to azimuthal asymmetries. Recent studies point to the importance of so-called trapped particles and the influence of asymmetries on them [79]. This somewhat subtle effect arises from the fact that particles trapped in electrostatic or magnetic wells (e.g., due to trap imperfections) do not experience the averaging effects that the bulk of the particles do, and so they can make larger radial excursions. When subsequently scattered out of this imperfection (i.e., trapping well), they can then cause greatly enhanced radial particle transport. From the perspective of antimatter-trap engineering, one typically relies upon empirical formulae based upon the now-extensive experimental studies. Data for the outward radial transport of particles (presumably due to asymmetries) from a plasma in a PM trap are shown in fig. 11 [80].

As shown in the figure, there are two regimes of plasma transport. At sufficiently high plasma densities, $\Gamma_0$ is independent of $n$, whereas at lower densities $\Gamma_0 \sim (nL)^2$. The transition between these two types of behavior appears to occur when the axial bounce frequency, $f_b = v_{th}/2L_p$ is approximately three times the Coulomb collision frequency [80]. However, there is no theory at present for this effect, and it is unclear whether this result will hold in other experiments. The values of $\Gamma_0$ shown in fig. 11 are among the smallest values reported for the given parameters. In other experiments, $\Gamma_0$ can be as much as an order of magnitude greater, presumably due to larger trap asymmetries [80]. In practice, the best one can do to estimate the outward transport (and/or confinement time) is to use the reported values as order of magnitude estimates of the outward transport.

In considering the effects on transport due to electric asymmetries that are static in the laboratory frame, they are expected to be largest in plasmas with a small rotation
frequency (i.e., in this case the transport is due to asymmetry-induced $E \times B$ flows). However at higher rotation frequencies, the rotation can also bring the asymmetry-induced fields (that are dc in the laboratory frame) into resonance with a plasma mode. This can act as a potent drag on the plasma and result in a high level of transport.

The $E \times B$ transport at low rotation frequencies has important consequences for the operation of buffer-gas traps. In the first stages of a buffer-gas trap, or in traps confining small numbers of positrons, the rotation frequency will be small because the positron density is low. Thus, for example, a small static electrostatic asymmetry (e.g., arising from patch-voltages on the electrodes) can induce the rapid dc $E \times B$ transport of the particles to the wall. In the case of the buffer gas trap, this means that one wants to get the particles out of these early trapping stages as quickly as possible and into the final stage where the plasma density (and hence the plasma rotation frequency) is higher. The small plasma rotation frequency in the first stage of buffer gas traps can potentially play a significant role in limiting the trapping efficiency of these devices.

4'4. Plasma heating. – Single-component plasmas in PM traps can be heated by various mechanisms, including ambient rf noise on the confining electrodes. One unavoidable heating source is the outward plasma expansion itself. Essentially, the radial, outward-directed electric field due to the plasma space charge preferentially gives the particles extra energy as they move outward radially. The heating rate, $\Gamma_h$, due to this effect can be written as [80-82],

$$\Gamma_h = \frac{1}{T} \frac{dT}{dt} = \left( \frac{e\phi_0}{2\eta T} \right) \Gamma_0,$$

where $1/\eta$ is the fraction of the space charge potential that is dropped across the plasma, assuming $\phi = \phi_0$ at $r = r_W$. For a rigid-rotor plasma with a constant radial density profile, $\eta = \left[ 1 + 2 \ln(a_w/r_p) \right]$, and $\phi_0$ represents the potential drop across the plasma itself. Note that the plasma potential can be quite large (tens of volts are not atypical), so that in modestly cold plasmas, it can be the case that $\Gamma_h \gg \Gamma_0$.

This heating must be mitigated by some type of cooling (e.g., cyclotron cooling or cooling due to collisions with gas molecules). In order for there to be a stable steady state, the heating rate must be smaller than the maximum cooling rate, i.e., $\Gamma_h/\Gamma_c < 1$, otherwise the temperature will increase in an uncontrolled manner.

4'5. If neutral collisions dominate both the transport and the cooling. – We can combine eqs. (10) and (11) to find for the heating rate

$$\Gamma_h = \frac{\nu_p}{4} \left( \frac{\omega_p}{\omega_c} \right)^2 \left( \frac{r_p}{\lambda_D} \right)^2.$$

Neutral collisions with molecules can provide cooling via the excitation of vibrations (e.g., as is the case for CF$_4$). Considering the excitation of a single level, the cooling rate is given by eq. (1). A measure of the effectiveness of this cooling can be obtained by
forming the ratio, $\beta$, of the heating rate given by eq. (12) to the cooling rate in eq. (1). Thus,

$$
\beta = \frac{\Gamma_h}{\Gamma_c} = \frac{\nu_p T}{4 \nu_j \dot{\varepsilon}_j} \left( \frac{\omega_p}{\omega_c} \right)^2 \left( \frac{r_p}{\lambda_D} \right)^2.
$$

As discussed above, the plasma temperature will be stable only for $\beta \leq 1$, and will “run away” for larger values of $\eta$, since $\beta \propto (n/B)^2$. This places an important constraint on the maximum achievable plasma density $n$.

It is useful to express the density in terms of the Brillouin limit density $n_B$ (i.e., the density at which $\omega_p^2 = \omega_c^2/2$; see eq. (19) and related discussion below for details), in which case

$$
\left( \frac{n}{n_B} \right)^2 = \frac{8 \nu_j \dot{\varepsilon}_j}{\nu_p T} \left( \frac{r_c}{r_p} \right)^2.
$$

Thus, to achieve high plasma densities, one would like a cooling gas with small $\nu_p$ and large $\nu_j$. Carbon tetrafluoride fits this bill. As discussed above, it has an unusually large value of $\nu_j$ (cf., fig. 7 [61]). It turns out that it also has a small value of $\nu_p$ [83], making it a good choice for this purpose.

We mention here two caveats to eqs. (12)–(14). Equation (14) is valid so long as the maximum density $n$ is not very close to $n_B$. Close to the Brillouin limit, the cycloidal $E \times B$ orbits of the particles are very large and nearly unconfined, and a more careful calculation (not done here) is required. A practical criterion might be to set the amplitude of the cycloidal motion $\delta r = E/\omega_c B$ to be $\leq 0.1 r_p$, which corresponds to $E/(\omega_c B r_p) \leq 0.1$ and $n/n_B \leq 0.1$. Further, we use particularly simple expressions for the collisional transport. Techniques such as Monte Carlo calculations would be very valuable in obtaining better estimates for the plasma expansion, heating and cooling [65].

### 4'6. Diagnostic techniques.

A variety of destructive and non-destructive techniques have been developed to measure the properties of non-neutral plasmas in traps, parameters such as plasma temperature, density, shape, and the total number of particles. Destructive diagnostics involve releasing the particles from the trap and detecting them in various ways. Absolute measurements of the total number of particles can be made by dumping the particles onto a collector plate and measuring the total charge [5]. In the case of positrons, the annihilation gamma rays can be detected when the particles are dumped, and the total particle number can thus be extracted using a calibrated detector. Radial profiles can be measured using a phosphor screen biased at a high voltage ($\sim 5$–10 kV). The resulting fluorescent light is measured using a charge coupled device (CCD) camera [84]. Plasma density can be inferred from the radial profiles and the total number of particles can be calculated using a Poisson-Boltzmann equilibrium code [85]. Plasma temperature can be measured by releasing particles slowly from the trap and measuring the tail of the particle energy distribution [86].
Destructive diagnostics have been employed extensively in the development of new techniques to manipulate and trap antiparticles. However, for experiments where the particles are collected for long times, such as antihydrogen production or the creation of giant pulses, destructive diagnostics are disadvantageous. Several non-destructive techniques have been developed, based on the properties of the plasma modes. For long cylindrical plasmas, the frequency of the diocotron mode yields the charge per unit length of the plasma, and hence provides information about the total number of particles [87,88].

For spheroidal plasmas in harmonic potential wells, the frequencies of the axial Trivelpiece-Gould modes [89] yield the aspect ratio of the plasma and can be used to measure plasma temperature in cases where the aspect ratio is constant [10-12,73,90]. Such a mode spectrum is shown in fig. 12. The total number of particles can be determined by the $Q$ factor of the response [12], or by independently calibrating the amplitude response [11,73]. Passive monitoring of thermally excited modes can also be used to determine the plasma temperature [91]. Driven-wave techniques have also been used to monitor positron plasmas used for antihydrogen production [10,12]. They were also applied to characterize electron plasmas that are used to trap and cool antiprotons [92].

5. Radial compression using rotating electric fields: the “rotating wall” technique

An important technique for manipulating non-neutral plasmas is to compress the plasma radially using a rotating electric field to apply a torque on the plasma. This is the so-called “rotating-wall” (RW) technique. It has provided important new capabilities for single-component plasma research, such as counteracting outward plasma transport and permitting essentially infinite confinement times. It was first used to
compress ion [93-96] and electron [81,97] plasmas. It has also been used to compress positron plasmas [6,8], including those for antihydrogen production [10,16,98] and for the brightness-enhancement of positron beams [13,14]. This RW technique was also an important facet of the first successful creation of the positronium molecule, $Ps_2$ [24]. It is expected to play a key role in planned work to produce giant pulses of positrons to create Bose-Einstein condensation (BEC) of positronium atoms and the stimulated emission of annihilation radiation [24].

The process of RW compression involves coupling a rotating electric field to the plasma to inject angular momentum. As described by eq. (4), this then reduces the second moment of the radial particle distribution [46]. The arrangement for RW compression is shown schematically in fig. 9. Phased sine waves applied to a sectored electrode are used to generate a rotating electric field with a low-order azimuthal mode number (e.g., $m_q = 1$) [6,8,9,80]. These fields produce a torque on the plasma, thereby compressing the plasma radially in a non-destructive manner.

Efficient cooling is required to counteract the heating caused by the torque-produced work done on the plasma. As described above, this cooling can be provided by cyclotron cooling (in the case of a strong confining magnetic field) [9,10,12,80], a buffer gas (in the case of a weak magnetic field) [6,8,99], or by sympathetic cooling using laser-cooled ions [94].

Early RW experiments relied on coupling to (Trivelpiece-Gould) plasma modes, which limited significantly the utility and flexibility of the technique. Two RW operating regimes were later discovered in which tuning to plasma modes is unnecessary. The first was in a plasma with buffer gas cooling when the plasma radius is comparable to the Debye length, $\lambda_D$ [8]. The second was in plasmas in a high-magnetic-field trap when the drive amplitude is sufficiently large (the “strong drive” regime) [80]. Most RW compression experiments now operate (or try to operate) in this second, strong-drive regime.

Shown in fig. 13 is an apparatus for studying PM plasmas cyclotron cooled in a high magnetic field. Shown in fig. 14 is an example of compression of an electron plasma in this device in the strong-drive regime. The protocol for these experiments is such that the RW is applied at fixed values of both $V_{RW}$ and $f_{RW}$. Above a certain drive amplitude, the plasma evolves to a high-density steady state in which $f_E \approx f_{RW}$ (cf. fig. 14). As illustrated in fig. 15, the radial density profiles of these plasmas are “flat-top” in shape (i.e., a constant-density rigid rotor in a state close to thermal equilibrium). Experiments at various values of $f_{RW}$ are shown in fig. 16, illustrating the ability to access a broad range of high-density states in this strong-drive regime.

The ability to access the strong drive regime depends upon overcoming the drag due to static asymmetries in the laboratory frame. These asymmetries drive waves (i.e., Trivelpiece-Gould modes) traveling backwards on the rotating plasma and thus act as a drag on it. This is illustrated in figs. 16 and 17 where a “step” appears in the data near the density $n = 0.4 \times 10^{19}$ cm$^{-3}$. The mode frequency is zero in the lab frame and referred to as a “zero frequency mode” (ZFM) [80]. The drag and drive torques on the plasma have been modeled to include this ZFM effect, [4], namely the total torque on
Fig. 13. – A high-magnetic field (5 T) UHV storage trap [100]. Also shown is a cut-away view of the electrode structure that contains two rotating-wall electrodes (left of center). The apparatus is also outfitted with a closed-cycle pulsed-tube refrigerator for cooling the electrodes.

Fig. 14. – Central electron density is shown as a function of time for various amplitudes of applied RW voltage at 6 MHz. Note the bifurcation from a low-density to a high-density state as $V_{RW}$ is increased above 0.7 V. Reprinted from ref. [80].
Fig. 15. – Radial profiles obtained by compression of an electron plasma in a 5 T magnetic field by the application of a rotating electric field at $f_{RW} = 6$ MHz beginning at $t = 0$ s. Steady-state compression is observed from $t = 10$ to 20 s, then the plasma is allowed to expand with the RW off. All profiles are close to thermal equilibrium, exhibiting flat-top profiles, except at $t = 2$ s, where the plasma is much hotter (i.e., $T \sim 3$ eV at that time). Reprinted from ref. [80]; see this reference for details.

Fig. 16. – Left: central plasma density following application of the RW at various frequencies at $V_{RW} = 1.0$ V; right: steady-state density as a function of applied RW frequency, following the transition to the high-density state. The step near $n = 0.4 \times 10^{10}$ cm$^{-3}$ is due to a so-called “zero-frequency” mode, which was key to understanding the high-density steady states. $B = 5$ T. Data from ref. [80].
Fig. 17. – Left: density as a function of $f_{RW}$, when $f_{RW}$ is fixed but the initial plasma density is smaller (upward arrow, ◊) or larger (downward arrow, ●) than that of the final, torque-balanced steady states (i.e., the stable fixed points); right: solutions of eq. (13) for $\tau = 0$, for the (●) stable and (○) unstable fixed points when approached varying $f_{RW}$ in the directions shown by the arrows. The model exhibits the same qualitative behavior as the data. Analysis from ref. [4]; see this reference for details.

The plasma will be

$$\tau = \eta \left( \frac{f_{RW} - f_E}{f_E} V_{RW}^2 \right) - \frac{\beta f_E}{D^2 + f_E^2} - \frac{\gamma \delta f_0}{(f_E - f_0)^2 + (\delta f_0)^2},$$

where $\eta$, $\beta$, $\gamma$ and $D$ are constants. The terms in eq. (15) represent the RW drive torque, $\tau_{RW}$ (first term) and the drag torques, $\tau_{drag}$. The latter is the sum of the second and third terms, namely the background drag torque (second term, coefficient $\beta$) on the plasma due to trap imperfections, and the drag due to the ZFM (third term, coefficient $\gamma$). The form of the second term was chosen empirically to model the observed outward transport data such as that shown in fig. 11. An example of the drag torque derived from that data is shown in fig. 18. The third term in eq. (15) is the ZFM drag term, which is modeled by a Lorentzian of width $\delta f_0$, centered at frequency $f_0$. Equilibrium is reached when $\tau = 0$, and this condition sets the plasma rotation frequency, $f_E$.

This model for the total torque on the plasma yields predictions that agree well with experimental observations [4]. It turns out that, for suitably strong drives to overcome the ZFM drag, the plasma spins up until $f_{RW} \approx f_E$ (which, in the language of nonlinear dynamics, is an “attracting fixed point” of eq. (15)). At lower values of $\tau_{RW}$, the plasma becomes “stuck” at a rotation frequency close to that of the ZFM (i.e., the “low-density fixed point” at $f_0$). The stable state to which the plasma relaxes depends upon which side of the ZFM the plasma starts: the fixed point is stable when $df_{RW}/df_{RW} > df_{drag}/df_{RW}$ and unstable when $df_{RW}/df_{RW} < df_{drag}/df_{RW}$. As a consequence, the plasma is predicted to exhibit hysteresis as a function of the RW drive amplitude.

As shown in fig. 17, the solutions to eq. (15) provide a good qualitative description of this hysteretic behavior and the high-density steady states that are achieved.
Similar hysteresis is also predicted and observed as a function of the rf drive voltage, $V_{RW}$ [4].

A key practical question is what limits the compression and the maximum achievable density. At UCSD, experiments are routinely conducted with relative ease up to $f_{RW} \sim 8$ MHz and spottily up to $\sim 18$ MHz. This limit may be due to spurious resonances in the electronic circuitry or perhaps something more fundamental (i.e., the inability to couple effectively to the plasma at high frequencies); this will require further study to resolve.

5'1. Rotating-wall compression in the single-particle regime. – Low-density positron gases in Penning traps (i.e., collections of particles outside the plasma regime) have also been compressed using the RW technique with gas cooling [99]. For successful RW operation it was necessary that the particles be confined in a harmonic electrostatic potential well in the direction of the confining, uniform magnetic field. As shown in fig. 19, good compression was observed when $f_{RW} \leq \omega_z$, where $\omega_z$ is the axial bounce frequency in the harmonic well. In this case, it is believed that the particles couple to a rotating particle bounce resonance. As shown in fig. 19, at frequencies above $\omega_z$, the particles are observed to heat rapidly and are de-confined.

The fact that the RW technique works in the single-particle regime is very useful in tailoring the charge clouds in buffer gas traps, particularly ones that operate with fewer stages. In such traps, the cycle time must be kept short to avoid outward radial transport and annihilation, and hence the positron density is relatively low (i.e., the trapped positrons are in the single-particle, non-plasma regime).

5'2. Heating due to rotating-wall compression. – Applying rotating electric fields to a plasma applies a torque $\tau_{RW}$ on it that heats the plasma by doing work on it. The heating rate can be written as [101]

\begin{equation}
    P_H = \omega_{RW} \tau_{RW},
\end{equation}

Fig. 18. – Drag torque $\tau_d$ as a function of plasma density $n_0$ derived from the expansion data shown in fig. 11 for $L_p = 24$ cm. Dashed lines are guides to the eye. This dependence of $\tau_d$ on $n_0$ motivated the specific form of the second term in eq. (15).
where $\omega_{RW}$ is the angular frequency of the rotating electric field. In the strong-drive regime, the minimum power input to the plasma will be when the drive and drag torques are in balance, in which case $\omega_{RW} \approx \omega_E$ and $P_H = \omega_E \tau_{RW}$, where $\omega_E$ is the angular rotation frequency of the plasma.

The asymmetry-induced drag torque $\tau_a$ can be obtained by relating the time derivative of the plasma angular momentum (cf. eq. (5)) to the outward expansion rate $\Gamma_0$ [80]. Assuming a plasma of $N$ particles with a flat-top density profile in surroundings at temperature $T_W$, the steady-state temperature $T$ will be [80]

$$T = T_W + \left( \frac{Ne^2}{3L_p} \right) \frac{\Gamma_0}{\Gamma_c}.$$

Illustrated in fig. 20 is the effect of plasma heating on RW compression. In this case the plasma is cooled by inelastic vibrational collisions with CF$_4$ molecules. Note that the temperature remains comparable to the $\nu_3$ mode energy of 0.16 eV (i.e., the dominant positron-impact vibrational excitation) over an order of magnitude increase in the RW voltage. When it does break away from this value, as the RW voltage is increased further, the temperature rises rapidly and the maximum achievable compression decreases quickly.

Good compression is obtained as long as the collisional excitation of the $\nu_3$ vibrational mode of CF$_4$ can control the plasma temperature. When the temperature increases much above the energy of this excitation ($\varepsilon_3 = 0.16$ eV), then the temperature runs away and the compression is much less efficient.

Note that the plasma temperature, given by eq. (17), is that expected for the minimum heating rate, which was obtained when the RW drive and asymmetry drag torque $\tau_a$ are
balanced in the strong-drive regime. If there is “slip” (i.e., if \( \omega_{RW} > \omega_E \)), the heating rate will be larger. In this case, the excess heating rate due to the slip will be

\[
\delta P = \tau_{RW} (\omega_{RW} - 2\pi f_E) = 2\pi \tau_{RW} \Delta f,
\]

where \( \Delta f \) is the so-called slip frequency.

5.3. Maximum achievable density using RW compression. – For many applications it is desirable to have as high a plasma density as possible. One constraint is the Brillouin limit. This limit arises from the fact that, for a particle in a PM trap rotating about the symmetry axis at frequency \( f_E \), the \( v \times B \) force acts both to provide the required inward centripetal force and to counteract the outward force due to the space-charge electric field. Due to the fact that the \( v \times B \) force is proportional to the particle velocity
$v$, and the centripetal force is proportional to $v^2$, this force balance is not possible above some maximum velocity $v$. And since the $E \times B$ rotation velocity, $v \propto n$, this imposes a maximum density limit, the so-called Brillouin limit.

The condition is [32]

\begin{equation}
\omega_p^2 = \omega_c^2 / 2,
\end{equation}

where $\omega_p$ is the plasma frequency. The resulting Brillouin density limit is

\begin{equation}
n_B [m^3] = 4.8 \times 10^{18} B^2[T],
\end{equation}

where $n_B$ is in units of $m^3$ and $B$ is in units of teslas. Above the Brillouin limit, particles at the plasma edge cannot be confined orbiting the axis of symmetry; they will move outward, unconfined.

However, if the plasma is in the presence of neutral-gas molecules, even below this limit, any scattering will cause the particles to make relatively large cycloid-like orbits, moving outward on each collision an average distance, $E/\omega_c B$, where $E$ is the space-charge electric field (cf. fig. 10). As the density increases, so will $E$, and hence the plasma will become more difficult to confine.

6. – Concluding remarks

The techniques described here have proven enormously useful in accumulating and manipulating positron, antimatter plasmas. They have played a central role in the quests to create low-energy antihydrogen and the positronium molecule, Ps$_2$. They have also proven crucial in studies of atomic physics processes such as positron scattering and annihilation in interactions with atoms and molecules.

That said, there are likely many opportunities for further improvement. While there are a myriad of possibilities, we mention here a few obvious ones. There are likely a number of ways to make buffer-gas positron traps simpler, more compact, and perhaps more efficient. This might be done by clever design of the neutral gas profile and the differential pumping arrangement. There are also questions as to what limits the maximum trapping efficiency and how this can be improved. Finally, the range of atomic and molecular gases explored for trapping and cooling, while extensive, has not been exhaustive; there may well be room for further improvement here too.

Regarding the RW technique, it is presently uncertain what limits the maximum density that can be achieved, and this is a crucial issue for many applications. One question is: can one approach the Brillouin limit, and if not, why not? There is also a question as to whether one might use a resonant structure to enhance greatly the cyclotron cooling [102]. If so, this likely will permit a broader range of operating parameters and the ability to operate at lower magnetic fields.
I would like to acknowledge the contributions of A. PASSNER, M. LEVENTHAL, T. J. MURPHY, M. TINKLE, R. G. GREAVES, J. R. DANIELSON, E. A. JERZEWSKI and T. M. O’NEIL to the work described here. I thank M. CHARLTON for his careful reading of the manuscript and helpful suggestions. This paper relies heavily upon the data and descriptions in refs. [2-4,82,103]. This work is supported by the U.S. NSF, grant PHY 07-13958.

REFERENCES


